Mechanics

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Preface

This is the first edition of our book on 'Mechanics' written for B.Sc. students of Indian Universities. The quality of teaching and training in any subject depends on the teaching skill of the teacher an on the quality of the text book available to the students. The present book on Mechanics is a very sincere effort to provide a high quality text book on the subject. Due to recent changes in the in the syllabus at the undergraduate level, it had become necessary to provide a new text book that gives a comprehensive coverage to the changed syllabus. The utility of this book is self-evident and the readers will be the best judges in this regard.

The principles of mechanics have been applied to three general realms of phenomena. The motions of such celestial bodies, planets, and satellites can be predicted with great accuracy thousands of years before they occur. (The theory of relativity predicts some deviations from the motion according to classical, or Newtonian, mechanics; however, these are so small as to be observable only with very accurate techniques, except in problems involving all or a large portion of the detectable universe.) As the second realm, ordinary objects on Earth down to microscopic size (moving at speeds much lower than that of light) are properly described by classical mechanics without significant corrections.

The subject matter of the present text book has been methodically and systematically developed from the fundamental laws of experimental physics. The treatments followed in this book are very simple, lucid and rather comprehensive. Great pain and care has been taken in explaining the obscure and difficult points in a simple language in a manner such that the

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students may feel no difficulty in following the subject.

It is hoped that this book will meet the need of advanced graduate students. I wish to thank to all the staff members of Buddha P. G. College, Kushinagar and friends for their help and cooperation. I gratefully acknowledge may indebtedness to various authors and publishers whose books have been freely consulted during the preparation of this book. I am also thankful to the publisher of this book for his keen interest and kind co-operation and printers for the nice printing. Lastly, I wish to thank all the persons who helped me directly or indirectly throughout this work. In spite of great care, several misprints and errors might have crept in. I will be grateful to those who point them out. Any constructive suggestions from teachers and students for the improvement of the book will be highly appreciated.

Kushinagar **Indrajeet Mishra**

Kartik Ekadashi 2020

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1

Introduction

Mechanics, science concerned with the motion of bodies under the action of forces, including the special case in which a body remains at rest. Of first concern in the problem of motion are the forces that bodies exert on one another. This leads to the study of such topics as gravitation, electricity, and magnetism, according to the nature of the forces involved. Given the forces, one can seek the manner in which bodies move under the action of forces; this is the subject matter of mechanics proper.

Historically, mechanics was among the first of the exact sciences to be developed. Its internal beauty as a mathematical discipline and its early remarkable success in accounting in quantitative detail for the motions of the Moon, the Earth, and other planetary bodies had enormous influence on philosophical thought and provided impetus for the systematic development of science into the 20th century.

Mechanics may be divided into three branches: statics, which deals with forces acting on and in a body at rest; kinematics, which describes the possible motions of a body or system of bodies; and kinetics, which attempts to explain or predict the motion that will occur in a given situation. Alternatively, mechanics may be divided according to the kind of system studied. The simplest mechanical system is the particle, defined as a body so small that its shape and internal structure are of no consequence in the given problem. More complicated is the motion of a system of two or more particles that exert forces on

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one another and possibly undergo forces exerted by bodies outside of the system.

The principles of mechanics have been applied to three general realms of phenomena. The motions of such celestial bodies as stars, planets, and satellites can be predicted with great accuracy thousands of years before they occur. (The theory of relativity predicts some deviations from the motion according to classical, or Newtonian, mechanics; however, these are so small as to be observable only with very accurate techniques, except in problems involving all or a large portion of the detectable universe.) As the second realm, ordinary objects on Earth down to microscopic size (moving at speeds much lower than that of light) are properly described by classical mechanics without significant corrections. The engineer who designs bridges or aircraft may use the Newtonian laws of classical mechanics with confidence, even though the forces may be very complicated, and the calculations lack the beautiful simplicity of celestial mechanics. The third realm of phenomena comprises the behaviour of matter and electromagnetic radiation on the atomic and subatomic scale. Although there were some limited early successes in describing the behaviour of atoms in terms of classical mechanics, these phenomena are properly treated in quantum mechanics.

Classical mechanics deals with the motion of bodies under the influence of forces or with the equilibrium of bodies when all forces are balanced. The subject may be thought of as the elaboration and application of basic postulates first enunciated by Isaac Newton in his *Philosophiae Naturalis Principia Mathematica* (1687), commonly known as the *Principia*. These postulates, called Newton's laws of motion, are set forth below. They may be used to predict with great precision a wide variety of phenomena ranging from the motion of individual particles to the interactions of highly complex systems.

In the framework of modern physics, classical mechanics can be understood to be an approximation arising out of the more profound laws of quantum mechanics and the theory of relativity.

However, that view of the subject's place greatly undervalues its importance in forming the context, language, and intuition of modern science and scientists. Our present-day view of the world and man's place in it is firmly rooted in classical mechanics. Moreover, many ideas and results of classical mechanics survive and play an important part in the new physics.

The central concepts in classical mechanics are force, mass, and motion. Neither force nor mass is very clearly defined by Newton, and both have been the subject of much philosophical speculation since Newton. Both of them are best known by their effects. Mass is a measure of the tendency of a body to resist changes in its state of motion. Forces, on the other hand, accelerate bodies, which is to say, they change the state of motion of bodies to which they are applied. The interplay of these effects is the principal theme of classical mechanics.

Although Newton's laws focus attention on force and mass, three other quantities take on special importance because their total amount never changes. These three quantities are energy, (linear) momentum, and angular momentum. Any one of these can be shifted from one body or system of bodies to another. In addition, energy may change form while associated with a single system, appearing as kinetic energy, the energy of motion; potential energy, the energy of position; heat, or internal energy, associated with the random motions of the atoms or molecules composing any real body; or any combination of the three. Nevertheless, the total energy, momentum, and angular momentum in the universe never changes. This fact is expressed in physics by saying that energy, momentum, and angular momentum are conserved. These three conservation laws arise out of Newton's laws, but Newton himself did not express them. They had to be discovered later.

It is a remarkable fact that, although Newton's laws are no longer considered to be fundamental, nor even exactly correct, the three conservation laws derived from Newton's laws—the conservation of energy, momentum, and angular momentum—

remain exactly true even in quantum mechanics and relativity. In fact, in modern physics, force is no longer a central concept, and mass is only one of a number of attributes of matter. Energy, momentum, and angular momentum, however, still firmly hold centre stage. The continuing importance of these ideas inherited from classical mechanics may help to explain why this subject retains such great importance in science today.

FUNDAMENTAL CONCEPTS OF MEHANICS

Units and dimensions

Quantities have both dimensions, which are an expression of their fundamental nature, and units, which are chosen by convention to express magnitude or size. For example, a series of events have a certain duration in time. Time is the dimension of the duration. The duration might be expressed as 30 minutes or as half an hour. Minutes and hours are among the units in which time may be expressed. One can compare quantities of the same dimensions, even if they are expressed in different units (an hour is longer than a minute). Quantities of different dimensions cannot be compared with one another.

The fundamental dimensions used in mechanics are time, mass, and length. Symbolically, these are written as t, m, and l, respectively. The study of electromagnetism adds an additional fundamental dimension, electric charge, or q. Other quantities have dimensions compounded of these. For example, speed has the dimensions distance divided by time, which can be written as l/t, and volume has the dimensions distance cubed, or l^3. Some quantities, such as temperature, have units but are not compounded of fundamental dimensions.

There are also important dimensionless numbers in nature, such as the number $\pi = 3.14159\ldots$. Dimensionless numbers may be constructed as ratios of quantities having the same dimension. Thus, the number π is the ratio of the circumference of a circle (a length) to its diameter (another length). Dimensionless numbers have the advantage that they are always the same, regardless of

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what set of units is being used. Governments have traditionally been responsible for establishing and enforcing standard units for the sake of orderly commerce, navigation, science, and, of course, taxation. Today all such units are established by international treaty, revised every few years in light of scientific findings. The units used for most scientific measurements are those designated the International System of Units (Système International d'Unités), or SI for short. They are based on the metric system, first adopted officially by France in 1795. Other units, such as those of the British engineering system, are still in use in some places, but these are now defined in terms of the SI units.

The fundamental unit of length is the metre. A metre used to be defined as the distance between two scratch marks on a metal bar kept in Paris, but it is now much more precisely defined as the distance that light travels in a certain time interval (1/299,792,458 of a second). By contrast, in the British system, units of length have a clear human bias: the foot, the inch (the first joint of the thumb), the yard (distance from nose to outstretched fingertip), and the mile (one thousand standard paces of a Roman legion). Each of these is today defined as some fraction or multiple of a metre (one yard is nearly equal to one metre). In the SI or the metric system, lengths are expressed as decimal fractions or multiples of a metre (a millimetre = one-thousandth of a metre; a centimetre = one-hundredth of a metre; a kilometre = one thousand metres).

Times longer than one second are expressed in the units seconds, minutes, hours, days, weeks, and years. Times shorter than one second are expressed as decimal fractions (a millisecond = one-thousandth of a second, a microsecond = one-millionth of a second, and so on). The fundamental unit of time (i.e., the definition of one second) is today based on the intrinsic properties of certain kinds of atoms (an excitationfrequency of the isotope cesium-133).

Units of mass are also defined in a way that is technically sound, but in common usage they are the subject of some

confusion because they are easily confused with units of weight, which is a different physical quantity. The weight of an object is the consequence of the Earth's gravity operating on its mass. Thus, the mass of a given object is the same everywhere, but its weight varies slightly if it is moved about the surface of the Earth, and it would change a great deal if it were moved to the surface of another planet. Also, weight and mass do not have the same dimensions (weight has the dimensions ml/t^2). The Constitution of the United States, which calls on the government to establish uniform "weights and measures," is oblivious to this distinction, as are merchants the world over, who measure the weight of bread or produce but sell it in units of kilograms, the SI unit of mass. (The kilogram is equal to 1,000 grams; 1 gram is the mass of 1 cubic centimetre of water—under appropriate conditions of temperature and pressure.)

Vectors

The equations of mechanics are typically written in terms of Cartesian coordinates. At a certain time t, the position of a particle may be specified by giving its coordinates $x(t)$, $y(t)$, and $z(t)$ in a particular Cartesian frame of reference. However, a different observer of the same particle might choose a differently oriented set of mutually perpendicular axes, say, $x2$, $y2$, and $z2$. The motion of the particle is then described by the first observer in terms of the rate of change of $x(t)$, $y(t)$, and $z(t)$, while the second observer would discuss the rates of change of $x2(t)$, $y2(t)$, and $z2(t)$. That is, both observers see the same particle executing the same motion and obeying the same laws, but they describe the situation with different equations. This awkward situation may be avoided by means of a mathematical construction called a vector. Although vectors are mathematically simple and extremely useful in discussing mechanics, they were not developed in their modern form until late in the 19th century, when J. Willard Gibbs and Oliver Heaviside(of the United States and Britain, respectively) each applied vector analysis in order to help express the new laws of electromagnetism proposed by James Clerk Maxwell.

A vector is a quantity that has both magnitude and direction. It is typically represented symbolically by an arrow in the proper direction, whose length is proportional to the magnitude of the vector. Although a vector has magnitude and direction, it does not have position. A vector is not altered if it is displaced parallel to itself as long as its length is not changed.

By contrast to a vector, an ordinary quantity having magnitude but not direction is known as a scalar. In printed works vectors are often represented by boldface letters such as A or X, and scalars are represented by lightface letters, A or X. The magnitude of a vector, denoted $|A|$, is itself a scalar—i.e., $|A| = A$.

Because vectors are different from ordinary (i.e., scalar) quantities, all mathematical operations involving vectors must be carefully defined. Addition, subtraction, three kinds of multiplication, and differentiation. There is no mathematical operation that corresponds to division by a vector.

If vector A is added to vector B, the result is another vector, C, written $A + B = C$. The operation is performed by displacing B so that it begins where A ends. C is then the vector that starts where Abegins and ends where B ends.

Vector addition is defined to have the (nontrivial) property $A + B = B + A$. There do exist quantities having magnitude and direction that do not obey this requirement. An example is finite rotations in space. Two finite rotations of a body about different axes do not necessarily result in the same orientation if performed in the opposite order.

Vector subtraction is defined by $A–B = A + (–B)$, where the vector $–B$ has the same magnitude as B but the opposition direction.

A vector may be multiplied by a scalar. Thus, for example, the vector $2A$has the same direction as A but is twice as long. If the scalar has dimensions, the resulting vector still has the same direction as the original one, but the two cannot be compared in magnitude. For example, a particle moving with constant

velocity v suffers a displacement s in time t given by $s = vt$. The vector v has been multiplied by the scalar t to give a new vector, s, which has the same direction as $v$but cannot be compared to v in magnitude (a displacement of one metre is neither bigger nor smaller than a velocity of one metre per second). This is a typical example of a phenomenon that might be represented by different equations in differently oriented Cartesian coordinate systems but that has a single vector equation (for all observers not moving with respect to one another).

The dot product (also known as the scalar product, or sometimes the inner product) is an operation that combines two vectors to form a scalar. The operation is written $A \cdot B$. If θ is the (smaller) angle between $A$and B, then the result of the operation is $A \cdot B = AB \cos \theta$. The dot product measures the extent to which two vectors are parallel. It may be thought of as multiplying the magnitude of one vector (either one) by the projection of the other upon it. If the two vectors are perpendicular, the dot product is zero.

The cross product (also known as the vector product) combines two vectors to form another vector, perpendicular to the plane of the original vectors. The operation is written $A \times B$. If θ is the (smaller) angle between A and B, then $|A \times B| = AB \sin \theta$. The direction of $A \times B$ is given by the right-hand rule: if the fingers of the right hand are made to rotate from A through θ to B, the thumb points in the direction of $A \times B$. The cross product is zero if the two vectors are parallel, and it is maximum in magnitude if they are perpendicular.

The derivative, or rate of change, of a vector is defined in perfect analogyto the derivative of a scalar: if the vector A changes with time t, then

$$\frac{dA}{dt} = \lim_{\Delta t \to 0} \frac{A(t + \Delta t) - A(t)}{\Delta t}$$

Before going to the limit on the right-hand side of equation (1), the operations described are vector subtraction [$A(t + \Delta t)$ –